

Chiral symmetry breaking in 3-flavor Nambu-Jona Lasinio model in magnetic background

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Abstract

Effect of magnetic field on chiral symmetry breaking in a 3-flavor Nambu Jona Lasinio (NJL) model at finite temperature and densities is considered here using an explicit structure of ground state in terms of quark and antiquark condensates. While at zero chemical potential and finite temperature, magnetic field enhances the condensates, at zero temperature, the critical chemical potential decreases with increasing magnetic field.

1. Introduction

The effect of high density and/or high temperature on the QCD vacuum has been a major theoretical and experimental challenge in the physics of strong interaction. In addition to the effects of high temperature and density, the effect of high magnetic field has also attracted recent attention of physicists recently. The motivation behind this study is the possibility of creating ultra strong magnetic fields in non central collisions at RHIC and LHC which are estimated to be of hadronic scale [1,2] of the order of $eB \sim 2m_\pi^2$ ($m_\pi^2 \simeq 10^{18}$ Gauss) at RHIC, to about $eB \sim 15m_\pi^2$ at LHC [2].

The studies of the effect of magnetic field on the vacuum structure of QCD has indicated that magnetic field acts as a catalyser of chiral symmetry breaking (CSB) [3,4]. On the other hand, it has been argued recently that there is an inverse magnetic catalysis at finite baryon chemical potential [5].

In this work, we investigate the effect of finite density and temperature on the vacuum structure in the context of CSB in a magnetic background using a variational method for 3-flavor NJL model.

2. Model interaction and gap equation

We shall consider hot and dense quark matter in a constant magnetic field \mathbf{B} in the z -direction which can be obtained from a electromagnetic vector potential given by $A_\mu(\mathbf{x}) = (0, 0, Bx, 0)$. Solving the Dirac equation we get the energy of the n -th Landau level given as $\epsilon_n^i = \sqrt{m_i^2 + p_z^2 + 2n|q_i|B} \equiv \sqrt{m_i^2 + |\mathbf{p}_i^2|}$. q_i is the electromagnetic charge of the quark of the i -th flavor with current quark mass m_i .

To study CSB, we consider a trial state with quark-antiquark pairs as

$$|\Omega\rangle = \exp\left(\sum_{n=0}^{\infty} \int d\mathbf{p}_{y,z} q_r^{i\dagger}(n, \mathbf{p}_{y,z}) a_{r,s}^i(n, p_z) f^i(n, \mathbf{p}_{y,z}) \tilde{q}_s^i(n, -\mathbf{p}_{y,z}) - h.c.\right) |0\rangle. \quad (1)$$

In the above ansatz for the ground state, $f^i(n, p_z)$ is a real function describing the quark antiquark condensates related to the vacuum realignment for chiral symmetry breaking. This will be determined from the extremization of the thermodynamic potential. The spin dependent structure $a_{r,s}^i$ is given by

$$a_{r,s}^i = \frac{1}{|\mathbf{p}_i|} \left[-\sqrt{2n|q_i|B} \delta_{r,s} - ip_z \delta_{r,-s} \right] \quad (2)$$

The ansatz of Eq.(1) is a Bogoliubov transformation of the vacuum state $|0\rangle$. Further the effect of temperature and density can be included through a thermal Bogoliubov transformation using thermofield dynamics. All these formulations are discussed in detail in reference [6].

The expectation value of the chiral order parameter for the i -th flavor for this ground state is given by

$$\langle \bar{\psi}_i \psi_i \rangle_{\beta, \mu} = - \sum_{n=0}^{\infty} \frac{N_c |q_i| B \alpha_n}{(2\pi)^2} \int dp_z \cos \phi^i (1 - n_-^i(k, \beta) - n_+^i(k, \beta)) \equiv -I_i, \quad (3)$$

where, $\alpha_n = (2 - \delta_{n,0})$ is the degeneracy factor of the n -th Landau level and n_- and n_+ are distribution functions of quarks and anti quarks respectively. We have defined $\phi^i = \phi_0^i - 2f_i$ with $\cos \phi_0^i = m_i/\epsilon_{ni}$ and $\sin \phi_0^i = |\mathbf{p}_i|/\epsilon_{ni}$ [6].

Now for calculating the thermodynamic potential, we consider the 3-flavor Nambu Jona Lasinio model including the Kobayashi-Maskawa-Hooft (KMT) determinant interaction term. The corresponding Hamiltonian density is given as

$$\begin{aligned} \mathcal{H} = & \psi^\dagger (-i\boldsymbol{\alpha} \cdot \nabla - qBx\alpha_2 + \gamma^0 \hat{m}) \psi - G_s \sum_{A=0}^8 [(\bar{\psi} \lambda^A \psi)^2 - (\bar{\psi} \gamma^5 \lambda^A \psi)^2] \\ & + K [\det_f [\bar{\psi}(1 + \gamma_5)\psi] + \det_f [\bar{\psi}(1 - \gamma_5)\psi]] \end{aligned} \quad (4)$$

Where $\hat{m} = \text{diag}_f(m_u, m_d, m_s)$ is the current quark mass matrix in the flavor space. We assume isospin symmetry with $m_u = m_d$. λ^A , $A = 1, \dots, 8$ denote the Gellman matrices acting in the flavor space and $\lambda^0 = \sqrt{\frac{2}{3}} \mathbf{1}_f$, $\mathbf{1}_f$ as the unit matrix in the flavor space. The determinant term $\sim K$ breaks $U(1)_A$ symmetry. If the mass term is neglected, the overall symmetry is $SU(3)_V \times SU(3)_A \times U(1)_V$. This spontaneously breaks to $SU(3)_V \times U(1)_V$ implying the conservation of the baryon number and the flavor number. The current quark mass term introduces additional explicit breaking of chiral symmetry leading to partial conservation of the axial current.

Now minimization of the thermodynamic potential for our model w.r.t the chiral condensate function $f_i(p_z)$ leads to the mass gap equation.

$$M_i = m_i + 4GI_i + 2K|\epsilon_{ijk}|I_jI_k \quad (5)$$

and the thermodynamic potential is given as

$$\begin{aligned} \Omega = & - \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^2} \int dp_z \omega_i + 2G \sum_i I_i^2 + 4K I_1 I_2 I_3 \\ & - \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^2 \beta} \int dp_z [\ln \{1 + e^{-\beta(\omega_i - \mu_i)}\} + \ln \{1 + e^{-\beta(\omega_i + \mu_i)}\}] \end{aligned} \quad (6)$$

where, $\omega_{i,n} = \sqrt{M_i^2 + p_z^2 + 2n|q_i B|}$ is the excitation energy with the “constituent quark mass” M_i . The zero temperature and the zero density contribution of the thermodynamic potential ($\Omega(T=0, \mu=0)$) in the above is ultraviolet divergent, which is also transmitted to the gap equation Eq.(5). To remove this divergence, we use a regularization procedure involving addition and subtraction of a divergent term which is discussed in detail in ref.[6].

3. Results and discussions

For numerical calculation, we choose the following parametrization of the hamiltonian given in Eq.(4). We take $m_u = m_d = 5.5$ MeV and $m_s = 0.1407$ GeV. The three momentum cut off for NJL model is $\Lambda = 0.6023$ GeV. The dimensionless couplings are $G_s \Lambda^2 = 1.835$ and $K \Lambda^5 = 12.36$. With these parameters, the vacuum masses of up and down quarks turn out to be 368 MeV and mass of the strange quark is 549 MeV. In Fig.

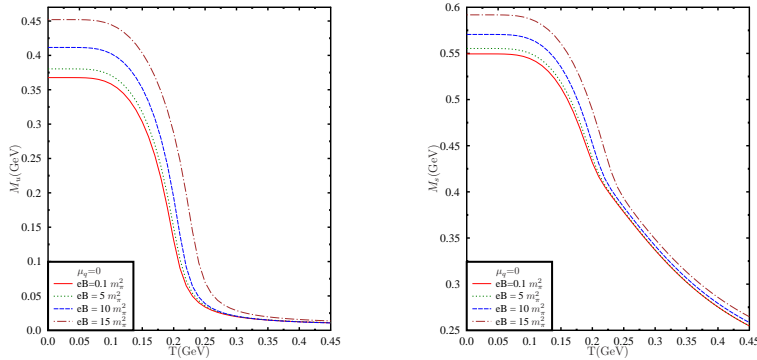


Fig. 1. Constituent quark masses with temperature for different magnetic fields. Figure in the left panel shows the mass of up quark M_u at zero baryon chemical potential as a function of temperature for different values of the magnetic field. Figure in the right panel shows the same for the strange quark mass M_s . Both the subplots correspond to nonzero values for the current quark masses given as $m_u=5.5$ MeV and $m_s=140.7$ MeV.

1, we have shown the effect of temperature on the constituent quark mass for different magnetic field strength at zero baryon chemical potential. The constituent quark mass smoothly approaches the current quark mass as temperature is increased. The magnetic field clearly enhances the chiral condensate. In Fig. 2, we show the effect of magnetic field and chemical potential on chiral symmetry breaking. As chemical potential is increased there is a first order transition with the order parameter changing discontinuously at the critical chemical potential μ_c . Although the condensate value increase with the magnetic field before the transition, the value of μ_c *decreases* with magnetic field. This phenomena is termed as inverse magnetic catalysis in Ref.[5] where chiral symmetry breaking has been considered in a holographic model. A more detailed discussion on the effect of magnetic field, temperature and density on chiral symmetry breaking, equation of state as well as charge neutral matter has been given in Ref.[6].

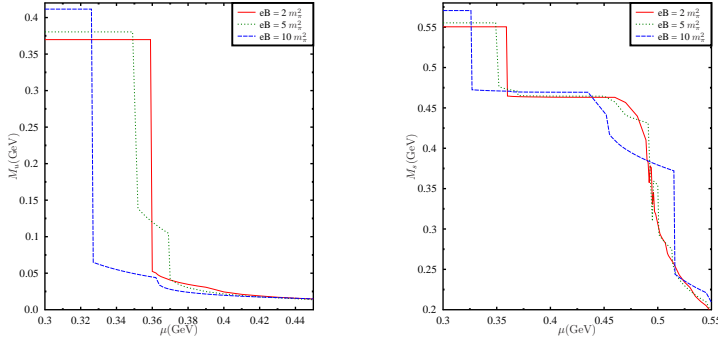


Fig. 2. Constituent quark masses as functions of μ_q for $T = 0$. The up quark masses are shown in left panel and figure in the right panel shows strange quark masses for different strength of magnetic fields.

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